Early Quantum Theory

Black-body Absorber: ____

incident radiation → all absorbed by black-body → all re-emitted by black-body

Late 19th Century picture...

Hertz/Maxwell: "EM Radiation is **waves**."
Michelson/Morley: "There is **no ether**." *...dichotomy?*

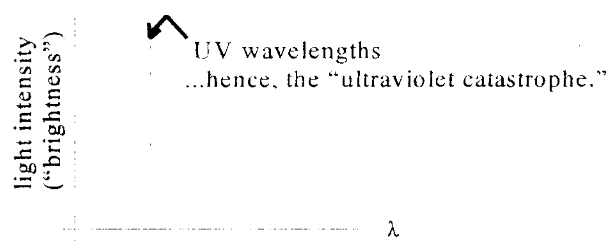
Observed black-body emission profile:

This had been measured and observed for years, and accepted as empirical fact.

Best theoretical model of EM emission in 1890s:

Lord Rayleigh + Sir James Jeans (England)

Function: $I(\lambda,T) = \dfrac{2\pi ckT}{\lambda^4}$



I: light intensity λ: wavelength (m) T: temp. (K) c: speed of light k: Boltzmann's constant

Also, on a more technical note, the area under the curve from the Rayleigh-Jeans expression will "diverge to infinity" since you can have any wavelength you want (i.e., $\int I(\lambda,T)d\lambda \rightarrow \infty$ for infinitesimal $d\lambda$).

So now we're stuck... EM radiation clearly behaves like waves, but we don't have an accurate way of predicting how they're emitted, or in what quantity.

MAX PLANCK TO THE RESCUE!!!

(pictured at right, playing the piano: from www.max-planck.mpg.de)

What was Planck's wacky idea in 1900?

____

In other words...

____

____

Planck's function, which *accurately* described the observed black-body emission: $I(\lambda,T) = \dfrac{2\pi hc^2}{\lambda^5(e^{hc/\lambda kT}-1)}$

h: "Planck's Constant" e: base of the "natural logarithm," 2.718... (cf. π)

"Big deal," you say. "So it's another goofy-looking function that describes light intensity. So what?"

- For long wavelengths, Planck's equation agrees with the Rayleigh-Jeans law (which itself was based on classical wave calculations).
- For short wavelengths, Planck's equation agrees with experimentally-observed radiation patterns.

Ground-breaking assumptions:

1. EM waves are not like other waves... they are little "packages" of pure energy called **QUANTA**. This is how we typically represent a quantum wave-packet:  (v=3×10^8 m/s)

   Hence... ***energy is transmitted from one place to another in discrete chunks***.

   Recall an electron vibrating up-and-down in a broadcast antenna, which gives off energy in the form of EM radiation. What does Planck's Assumption #1 imply?

   ____

   ____

   photon: ____________________________________

   Food for thought: Compare this to the standing wave patterns seen in vibrating strings.
   Jot down a couple of quick ideas in the space below.

2. The energy contained in a quantum is related to its frequency:

$$\boxed{E_{\text{quantum}} = hf}$$

E: energy (J) h: Planck's constant (6.626×10^{-34} J·s) f: frequency (Hz, or s^{-1})

** Frequency is sometimes given the symbol ν, the Greek letter "nu" (cf. the Roman letter "n")... then, $E=h\nu$. Same equation, though. (Yes, it looks like the letter "v.")

** Remember, one electron-volt is a unit of energy such that 1 eV = 1.60×10^{-19} J. Find the value of Planck's constant in units of eV·s.

Reference:
Serway, R.A. *Physics for Scientists and Engineers, 3rd Edition, updated.* Orlando, Florida: Harcourt Brace Jovanovich (1992), pp. 1147-1149.

SPH4U PLANCK'S QUANTUM HYPOTHESIS

SAMPLE problem 1

Calculate the energy in joules and electron volts of

(a) a quantum of blue light with a frequency of 6.67×10^{14} Hz

(b) a quantum of red light with a wavelength of 635 nm

Practice

Understanding Concepts

1. Explain which of the following quantities are discrete: time, money, matter, energy, length, scores in hockey games.
2. Determine the energy, in electron volts, for quanta of electromagnetic radiation with the following characteristics:
   (a) wavelength = 941 nm (infrared radiation)
   (b) frequency = 4.4×10^{14} Hz (red light)
   (c) wavelength = 435 nm (violet light)
   (d) frequency = 1.2×10^{18} Hz (X rays)
3. Calculate the wavelength, in nanometres, of a quantum of electromagnetic radiation with 3.20×10^{-19} J of energy. What colour is it? (See Section 9.6 for reference.)
4. Calculate the frequency of a 2.25-eV quantum of electromagnetic radiation.
5. Compare the respective energies of a quantum of "soft" ultraviolet radiation ($\lambda = 3.80 \times 10^{-7}$ m) and a quantum of "hard" ultraviolet radiation ($\lambda = 1.14 \times 10^{-7}$ m), expressing your answer as a ratio.

Making Connections

6. As you read this text, your body may be bombarded with quanta from radio waves ($\lambda = 10^2$ m) and quanta of cosmic rays, energetic particles, rather than electromagnetic waves, which nevertheless prove in quantum theory to have a wave aspect ($\lambda = 10^{-16}$ m). How many quanta of radio waves would it take to impart the same amount of energy as a single quantum of cosmic radiation? Comment on the relative biological hazards posed by these two sources of energy.

Answers

2. (a) 1.32 eV
   (b) 1.8 eV
   (c) 2.86 eV
   (d) 5.0×10^3 eV
3. 622 nm
4. 5.43×10^{14} Hz
5. 3.3:1
6. 10^{18}